WEEKLY TEST MEDICAL PLUS -02 TEST - 10 Balliwala
SOLUTION Date 08-09-2019

[PHYSICS]

1. Kepler's second law is a consequence of conservation of angular momentum

2. According to Kepler's first law, every planet moves in an elliptical orbit with the sun situated at one of the foci of the ellipse.

   In options (a) and (b) sun is not at a focus while in (c) the planet is not in orbit around the sun. Only (d) represents the possible orbit for a planet.

3. Kepler's law $T^2 \propto R^3$

4. During path *DAB* planet is nearer to sun as comparison with path *BCD*. So time taken in travelling *DAB* is less than that for *BCD* because velocity of planet will be more in region *DAB*.

5. Time period of a revolution of a planet,

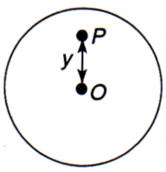
$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{GM_S}{r}}} = \frac{2\pi r^{3/2}}{\sqrt{GM_S}}$$

6. Gravitational force is independent of the medium. Thus, gravitational force will be same i.e., *F*.

7. In arrangement 1, both forces act in the same direction. In arrangement 3, both the forces act in opposite direction. This alone decides in favour of option (a),

8. If a point mass is placed inside a uniform spherical shell, the gravitational force on the point mass is zero. Hence, the gravitational force exerted by the shell on the point mass is zero.

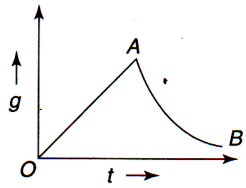
9. $g_d = g\left(1 - \frac{d}{g}\right)$



or $\quad g_d = g\frac{R-d}{R}$

or $\quad g_d = \frac{gy}{R}$ or $g_d \propto y$

So, within the Earth, the acceleration due to gravity varies linearly, with the distance from the centre of the Earth. This explains the linear portion *OA* of the graphs.



10. The value of g at the height h from the surface of earth

$$g' = g\left(1 - \frac{2h}{R}\right)$$

The value of g at depth x below the surface of earth

$$g' = g\left(1 - \frac{x}{R}\right)$$

These two are given equal, hence $\left(1 - \frac{2h}{R}\right) = \left(1 - \frac{x}{R}\right)$

On solving, we get $x = 2h$

11. Acceleration due to gravity $g = \frac{4}{3}\pi\rho GR \;\therefore\; g \propto \rho R$

or $\frac{g_m}{g_e} = \frac{\rho_m}{\rho_e} \cdot \frac{R_m}{R_e}$

$$\left[\text{As } \frac{g_m}{g_e} = \frac{1}{6} \text{ and } \frac{\rho_e}{\rho_m} = \frac{5}{3} \text{(given)}\right]$$

$$\therefore \quad \frac{R_m}{R_e} = \left(\frac{g_m}{g_e}\right)\left(\frac{\rho_e}{\rho_m}\right) = \frac{1}{6} \times \frac{5}{3} \therefore R_m = \frac{5}{18}R_e$$

12. Acceleration due to gravity $g = \frac{GM}{R^2}$

$$\therefore \quad \frac{g_{moon}}{g_{earth}} = \frac{M_{moon}}{M_{earth}} \cdot \frac{R_{earth}^2}{R_{moon}^2} = \left(\frac{1}{80}\right)\left(\frac{4}{1}\right)^2$$

$$g_{moon} = g_{earth} \times \frac{16}{80} = \frac{g}{5}$$

13. Acceleration due to gravity $g = \frac{4}{3}\pi\rho GR$

$$\therefore \quad g_1 : g_2 = R_1\rho_1 : R_2\rho_2$$

14. $g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{g}{4} = g\left(1 - \frac{d}{R}\right) \Rightarrow d = \frac{3R}{4}$

15. We know $g = \frac{GM}{R^2} = \frac{GM}{(D/2)^2} = \frac{4GM}{D^2}$

If mass of the planet $= M_0$ and diameter of the planet $= D_0$. Then $g = \frac{4GM_0}{D_0^2}$.

16. $\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2 = \left(\frac{R}{R+2R}\right)^2 = \frac{1}{9} \quad \therefore g' = \frac{g}{9}$.

17. Acceleration due to gravity on earth is

$$g = \frac{GM_E}{R_E^2} \qquad \text{(i)}$$

As $\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3} \Rightarrow M_E = \rho \frac{4}{3}\pi R_E^3$

Substituting this value in Eq. (i), we get

$$g = \frac{G\left(\rho \frac{4}{3}\pi R_E^3\right)}{R_E^2} = \frac{4}{3}\pi\rho G R_E \text{ or } \rho = \frac{3g}{4\pi G R_E}$$

18. $g = \frac{GM}{R^2}$

$$\frac{\Delta g}{g} \times 100 = 2\frac{\Delta R}{R} \times 100 = 2 \times 1\% = 2\%$$

19. Gravitational P.E. = $m \times$ gravitational potential

$U = mV$

So the graph of U will be same as that of V for a spherical shell.

20. $$\Delta U = U_2 - U_1 = \frac{mgh}{1+\frac{h}{R_e}} = \frac{mgR_e}{1+\frac{R_e}{R_e}} = \frac{mgR_e}{2}$$

$$\Rightarrow \quad U_2 - (-mgR_e) = \frac{mgR_e}{2} \Rightarrow U_2 = -\frac{1}{2}mgR_e$$

21. $\Delta K.E. = \Delta U$

$$\frac{1}{2}MV^2 = GM_e M\left(\frac{1}{R} - \frac{1}{R+h}\right) \qquad \text{(i)}$$

Also $g = \frac{GM_e}{R^2}$ (ii)

On solving (i) and (ii) $h = \frac{R}{\left(\frac{2gR}{V^2} - 1\right)}$

22. Potential energy $U = \frac{-GMm}{r} = -\frac{GMm}{R+h}$

$$U_{\text{initial}} = -\frac{GMm}{3R} \text{ and } U_{\text{final}} = -\frac{-GMm}{2R}$$

Loss in PE = gain in $KE = \frac{GMm}{2R} - \frac{GMm}{3R} = \frac{GMm}{6R}$

23. $v = -\frac{GMM}{R} = -\frac{GM^2}{R}$

24. Before collision, $PE = mV = -\frac{GMm}{r}$

After collision, velocity will be zero. The wreckage will come to rest. The energy will be only potential energy.

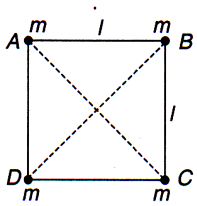
$$PE = -\frac{GMm}{r} = -\frac{2GMm}{r} \text{ Ratio} = 1/2$$

25. $\Delta u = -\left(\frac{GMm}{R+h}\right) - \left(-\frac{GMm}{R}\right)$

$H = 3R$

$\Delta u = -\frac{GMm}{4R} + \frac{GMm}{R} = \frac{3GMm}{4R^2} \times R$

$\Delta u = \frac{3}{4} mgR$

26. 

From figure

$AB = BC = CD = AD = l$

27. $U = \frac{-GMm}{r}$, $K = \frac{GMm}{2r}$ and $E = \frac{-GMm}{2r}$

For a satellite U, K and E vary with r and also U and E remain negative whereas K remains always positive.

28. $v = \sqrt{\frac{GM}{r}}$ if $r_1 > r_2$ then $v_1 < v_2$

Orbital speed of satellite does not depend upon the mass of the satellite.

29. $v = \sqrt{\frac{GM}{R+h}}$

For first satellite $h = 0$, $v_1 = \sqrt{\frac{GM}{R}}$

For second satellite $h = \frac{R}{2}$, $v_2 = \sqrt{\frac{2GM}{3R}}$

$$v_2 = \sqrt{\frac{2}{3}} v_1 = \sqrt{\frac{2}{3}} v$$

30. Since the planet is at the centre, the focus and centre of the elliptical path coincide and the elliptical path becomes circular and the major axis is nothing but the diameter. For a circular path:

$$\frac{mv^2}{r} = \sqrt{\frac{GM}{r^2}}m$$

Also $T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{GM}} \Rightarrow T^2 = \frac{4\pi^2 r^3}{GM}$

$\Rightarrow \quad r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \text{Radius}$

$\Rightarrow \quad \text{Diameter (major axis)} = 2\left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$

31. Orbital velocity of the satellite is

$v = \sqrt{\frac{GM_E}{r}}$ where M_E is the mass of the earth

Kinetic energy, $K = \frac{1}{2}mv^2 = \frac{GM_E m}{2r}$

where m is the mass of the satellite.

$$K \propto \frac{1}{r}$$

Hence, option (b) in incorrect.

Linear momentum, $p = mv = m\sqrt{\frac{GM_E}{r}}$

$$p \propto \frac{1}{\sqrt{r}}$$

Hence, option (c) is incorrect.

Frequency of revolution, $\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{GM_E}{r^3}}$

$$\nu \propto \frac{1}{r^{3/2}}$$

Hence, option (d) is correct.

32. Time period, $T = \frac{2\pi R}{\sqrt{\frac{GM_m}{R}}} = \frac{2\pi R^{3/2}}{\sqrt{GMm}}$

where the symbols have their meanings as given. Squaring both sides, we get

$$T^2 = \frac{4\pi^2 R^3}{GMm}$$

33. Total energy of the orbiting satellite of mass m having orbital radius r is

$E = -\frac{GMm}{2r}$ where M is the mass of the planet.

Additional kinetic energy required to transfer the satellite from a circular orbit of radius R_1 to another radius R_2 is

$$= E_2 - E_1$$
$$= -\frac{GMm}{2R_E} - \left(-\frac{GMm}{2R_1}\right) = -\frac{GMm}{2R_2} + \frac{GMm}{2R_1}$$
$$= \frac{GMm}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

34. Total energy of orbiting satellite at a height h is

$$E = -\frac{GM_E m}{2(R_E + h)}$$

The total energy of the satellite at infinity is zero.

$\therefore$ Energy expended to rocket the satellite out of the earth's gravitational field is

$$\Delta E = E_\infty - E$$
$$= 0 - \left(-\frac{GM_E m}{2(R_E + h)}\right) = \frac{GM_E m}{2(R_E + h)}$$

35. Let m_1 is mass of core and m_2 is of outer portion

$$m_1 = \frac{4}{3}\pi R^3 \rho_1, \quad m_2 = \frac{4}{3}\pi[(2R)^3 - R^3]\rho_2$$

Given that: $\frac{Gm_1}{R^2} = \frac{G(m_1 + m_2)}{(2R)^2} \Rightarrow \frac{\rho_1}{\rho_2} = \frac{7}{3}$

36. $v_1 r_1 = v_2 r_2$

$$\frac{KE_1}{KE_2} = \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \left(\frac{v_1}{v_2}\right)^2 = \left(\frac{r_2}{r_1}\right)^2$$

37 For two points on same orbit $L = mv_A r_A = mvr_B$

$$v_A = \frac{vr_B}{r_A} \qquad \text{(i)}$$

For two points on different orbits.

$$v = \sqrt{\frac{GM}{r}} \quad \frac{v_0}{v_A} = \left(\frac{r_A}{1.2r_A}\right)^{1/2}$$
$$v_0 = v_A\left(\frac{r_A}{1.2r_A}\right)^{1/2} = \frac{vr_B}{r_A}\left(\frac{r_A}{1.2r_A}\right)^{1/2} = \frac{vr_B}{r_A\sqrt{1.2}}$$

38. $\frac{1}{2}mv_{min}^2 = \left[-\frac{GMm}{r} - \frac{GMm}{r}\right]$

$$-\left[-\frac{GMm}{(2r-a)} - \frac{GMm}{a}\right]$$

$$= \frac{2GMm(a^2 - 2ar + r^2)}{ar(2r-a)}$$

or $\quad v_{min} = \sqrt{\frac{GM}{a}} \times \frac{2(r-a)}{[r(2r-a)]^{1/2}}$

So, $\quad K = \frac{2(r-a)}{[r(2r-a)]^{1/2}}$

39. $m_1 r_1 = m_2 r_2\ r_1 = \frac{m_2 r}{m_1 + m_2}$ (i)

$$m_1 r_1 \omega^2 = \frac{Gm_1 m_2}{r^2}\ \omega^2 = \frac{G(m_1 + m_2)}{r^3}$$

[From (i)] or $r = \left[\frac{G(m_1 + m_2)}{\omega^2}\right]^{1/3}$

$$(m_1 + m_2)^{1/3} = 2m_1 + m_2 = 8$$

and $\quad m_2 - m_1 = 6 \qquad$ (given)

which gives $m_1 = 1$ and $m_2 = 7$ units

$$\frac{m_1}{m_2} = \frac{1}{7}$$

40. Interstellar velocity $v' = \sqrt{\frac{GM}{r}} = R\sqrt{\frac{g}{(R+h)}}$

$$= \sqrt{v^2 - v_e^2}$$

where v = projection velocity

$$\frac{R^2 g}{(r+h)} = v^2 - 2gR \text{ Solving } v^2 = \frac{23gR}{11}$$

41. For observer, $T' = \frac{2\pi}{\omega_S - \omega_E} = \frac{T_S T_E}{T_E - T_S}$

$= T_E$ (given) or, $T_E^2 = 2T_S T_E\ \ T_S = T_E/2$

42. Time period is minimum for the satellites with minimum radius of the orbit i.e. equal to the radius of the planet. Therefore.

$$\frac{GMm}{R^2} = \frac{mv^2}{R} \Rightarrow V = \sqrt{\frac{GM}{R}}$$

$$T_{\min} = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi R\sqrt{R}}{\sqrt{GM}}$$

using $M = \frac{4}{3}pR^3 \cdot \rho$ $\quad T_{\min} = \sqrt{\frac{3\pi}{G\rho}}$

Using values $T_{\min} = 3000$ s

43. Conserving angular momentum

$m \cdot (V_1 \cos 60°) \cdot 4R = m \cdot V_2 \cdot R;\ \frac{V_2}{V_1} = 2$ Conserving energy of the system

$$-\frac{GMm}{4R} + \frac{1}{2}mV_1^2 = -\frac{GMm}{R} + \frac{1}{2}mV_2^2$$

$$\frac{1}{2}V_2^2 - \frac{1}{2}V_1^2 = \frac{3}{4}\frac{GM}{R}$$

or $\quad V_1^2 = \frac{1}{2}\frac{GM}{R}$

$$V_1 = \frac{1}{\sqrt{2}}\sqrt{64 \times 10^6} = \frac{8000}{\sqrt{2}}\text{ m/s}$$

44. The time period of satellite, $T \propto r^{3/2}$

or $\quad T \propto (R_e + h)^{3/2}$

For a satellite revolving close to surface of earth's $h = 0$

$\therefore\ T \propto R_e^{3/2}$. It is evident that the period of revolution of a satellite depends upon its height above the earth's surface. Greater is the height of a satellite above the earth's surface greater is its period of revolution.

45. Geostationary satellites orbit around the earth in the equatorial plane with time period of 24 hours. Since the earth rotates with the same period, the satellite would appear fixed from any point on earth.